nonreciprocal optical IC devices possessing various functions would be constructed by using the perturbed systems treated in the present paper.

V. Conclusion

The coupled optical waveguides, consisting of two isotropic dielectric slab waveguides coupled through anisotropic or gyrotropic materials inserted between them, have been treated theoretically in detail. It has been found that the *AL*- and *AP*-perturbation systems show the reciprocal mode conversion, while the *GL*- and *GP*-perturbation systems possess the property of nonreciprocal mode conversion, and the *GE*-perturbation system causes nonreciprocal phase shift for the TM_0 mode, whereas the *AE*-perturbation system shows no influence upon both TE_0 and TM_0 modes. As an example of application of these perturbed systems, the nonreciprocal optical IC mode converter has been proposed, and the numerical design example of the optical IC circulator has been given. This circulator requires no mode separators at both input and output ports. This circulator can also be utilized as an isolator without using mode filters at both input and output ports. In order to realize these devices, the progress of the fabrication techniques, together with the development of magnetooptic materials which possess a large Faraday effect, is required.

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Linear Power Responses of an Optical Fiber

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***Abstract*—It is known that an optical fiber behaves linearly in terms of power when the modulation frequency is smaller than the spectrum width of the light source. In order to calculate the impulse or frequency power responses with a modal calculation, it is shown that the powers carried by the different modes are independent in usual cases. Different formulas are proposed for the linear responses when there is no mode coupling, and the corresponding validity conditions are given.**

I. Introduction

A DESIRABLE characteristic of any transmission system is the linear relation between the output and input variables. In the case of transmission through optical fibers, the output variable is the current generated by the photodetector, and it is proportional to the optical power. Then the fiber must be linear in terms of power. Some aspects of this linearity have already been studied [1], [2]. It may be obtained by using an incoherent source of spectral width $\Delta\nu$ when the modulation frequencies are quite lower than $\Delta\nu$ [1].

A modal calculation of the impulse and frequency power responses, when there is no mode coupling, is proposed (Section III). But before exposing our results, we must justify the validity of such a method (Section II).

II. Do Different Modes Carry Independent Contributions to the Guided Power?

It is commonly assumed that the answer is positive.

Since powers of unmodulated modes are independent in case of lossless guides only, we shall consider our fiber as a

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lossless guide (which actually is a very good approximation). In order to have a simple theory, we shall neglect mode coupling.

We shall call ν_0 the center frequency of the light source. At any frequency ν, we may define normal modes (k), with β_k as propagation constant, and $(\boldsymbol{e}_k,\boldsymbol{h}_k)$ as electric and magnetic transverse components; $\boldsymbol{e}_k$ is real; $\boldsymbol{h}_k$ and β_k are real or imaginary; these components depend on frequency with $\boldsymbol{e}_k(-\nu) = \boldsymbol{e}_k(\nu)$, $\boldsymbol{h}_k(-\nu) = \boldsymbol{h}_k$, and $\beta_k(-\nu) = -\beta_k(\nu)$.

The orthogonality relation is [3]

$$\{\boldsymbol{e}_k \times \boldsymbol{h}_p^* \cdot \boldsymbol{u}\} = N_p\delta_{pk}. \quad (1)$$

($\{\cdots\}$ is the integration over the cross section; $\boldsymbol{u}$ is the unit vector along the propagation axis (z axis); N_p is the normalization constant, equal to unity for propagating modes.)

We shall denote $\beta_k{}^0$, $\boldsymbol{e}_k{}^0$, $\boldsymbol{h}_k{}^0, \cdots$ the quantities related to the mode (k) at the frequency ν_0.

The power guided by the fiber at the abscissa z is

$$P = \{\boldsymbol{E}_T \times \boldsymbol{H}_T \cdot \boldsymbol{u}\} \quad (2)$$

where $\boldsymbol{E}_T$, $\boldsymbol{H}_T$ are the transverse components of the field at z. They are real quantities, and their general expressions are

$$\begin{pmatrix}\boldsymbol{E}_T\\ \boldsymbol{H}_T\end{pmatrix} = \int_0^\infty d\nu \sum_k \xi_k(\nu) \begin{pmatrix}\boldsymbol{e}_k\\ \boldsymbol{h}_k\end{pmatrix} \exp i(2\pi\nu t - \beta_k z) + \text{c.c.} \quad (3)$$

where the modal amplitudes $\xi_k(\nu)$ are independent from z since we assumed no mode coupling. In order to simplify the following development, we shall only deal with forward fields, i.e., we limit the summation $\sum_k$ to forward modes.

At first sight, (3) does not lead to a simple expression of the power. The contributions of the individual modes are not clear before one notices that the modal amplitudes can be neglected everywhere except in a narrow frequency range centered on ν_0, where one may likely ignore the variations of $(\boldsymbol{e}_k,\boldsymbol{h}_k)$ and replace them by $(\boldsymbol{e}_k{}^0,\boldsymbol{h}_k{}^0)$. Before using this approximation, we may still be rigorous and expand the guided field in terms of modes $(\boldsymbol{e}_k{}^0,\boldsymbol{h}_k{}^0)$:

$$\boldsymbol{E}_T = \sum_k c_k'(z,t)\boldsymbol{e}_k{}^0 + \text{c.c.} \quad \boldsymbol{H}_T = \sum c_k''(z,t)\boldsymbol{h}_k{}^0 + \text{c.c.} \quad (4a)$$

with

$$N_k{}^0 c_k' = \int_0^\infty d\nu \sum_p \{\boldsymbol{e}_p \times \boldsymbol{h}_k{}^{0*} \cdot \boldsymbol{u}\}\xi_p(\nu) \cdot \exp i(2\pi\nu t - \beta_p z) \quad (4b)$$

$$N_k{}^{0*} c_k'' = \int_0^\infty d\nu \sum_p \{\boldsymbol{e}_k{}^{0*} \times \boldsymbol{h}_p \cdot \boldsymbol{u}\}\xi_p(\nu) \cdot \exp i(2\pi\nu t - \beta_p z). \quad (4c)$$

The properties of modal functions and the orthogonality relation allow us to write the power as

$$P = \text{Re} \sum_k (c_k' c_k''^* N_k \pm c_k' c_k'' N_k). \quad (5)$$

This relation seems to demonstrate the separate contributions of the different modes to the power. But it does not, since, from (4b) and (4c), the (c_k') and (c_k'') are related to all the modes at the same time.

The aforementioned approximation consists of putting $(\boldsymbol{e}_p,\boldsymbol{h}_p) = (\boldsymbol{e}_p{}^0,\boldsymbol{h}_p{}^0)$ in (4), which leads to

$$c_k' \simeq c_k'' \simeq c_k = \int_0^\infty \xi_k(\nu) \exp i(2\pi\nu t - \beta_k z)\, d\nu. \quad (6)$$

Then, c_k' and c_k'' are related to the mode (k) only and the mode powers are independent. In order to evaluate the accuracy of this approximation, we made a rough calculation of the errors done on c_k' and c_k'' (Appendix A). We found

$$\left|\frac{c_k' - c_k}{c_k}\right| \simeq \left|\frac{c_k'' - c_k}{c_k}\right| \sim \frac{\delta\nu}{4\nu_0}\left(1 - \frac{v_\phi}{v_g}\right) \quad (7)$$

where $\delta\nu$ is the width of the domain in which the mode (k) is significantly excited (practically, two or three times the spectrum width $\Delta\nu$) and where v_ϕ and v_g are the phase velocity and the group velocity, respectively. The ratio $\Delta\nu/\nu_0$ is rather small for light sources, though not always very small (3.10^{-3}–3.10^{-5} for GaAs laser diodes, but only 3.10^{-2} for LED). The second term $(1 - v_\phi/v_g)$ is small for usual fibers, where there is a little difference between the core refractive index n_1 and the cladding index n_2. For instance, with parabolic graded index fibers, v_g is c/n_1 and v_ϕ varies from c/n_1 to c/n_2 [4] and then

$$\left|1 - \frac{v_\phi}{v_g}\right| < \left|\frac{n_1 - n_2}{n_2}\right|.$$

Roughly, this ratio is 10^{-2} in usual fibers. One obtains similar results in step index fibers. Then we conclude that in practical cases, we may replace c_k' and c_k'' by c_k with a high level of accuracy and thus, the modulated modes are independent in power.

III. Calculation of the Impulse and Frequency Power Responses

We may then calculate the response functions by adding the response functions of individual modes. We write the input field in mode (k) delivered by a modulated incoherent source as

$$(\boldsymbol{E},\boldsymbol{H}) = M(t)f(t)(\boldsymbol{e}_k{}^0,\boldsymbol{h}_k{}^0) \quad (8)$$

where $M(t)$ is a certain function corresponding to the modulation and $f(t)$ is a random stationary function describing the source. It is possible to obtain the ξ_k and the propagating power directly from (8) and (6), but we prefer to expose a simpler derivation. The propagating field is a linear function of the input field (8):

$$c_k(z,t) = \int_{-\infty}^{+\infty} l_k(t - \tau)M(\tau)f(\tau)\, d\tau \quad (9)$$

where l_k is the impulse field response over the length z. From (6), it is the Fourier transform of $\exp - i\beta_k(\nu)z$. With an incoherent source, the measured power is the random average

$$\langle c_k{}^2\rangle = \int\int l_k(t - \tau)l_k(t - \tau + \theta)M(\tau)M(\tau - \theta) \cdot \langle f(\tau)f(\tau - \theta)\rangle\, d\tau\, d\theta.$$

There appears the correlation function $\Gamma(\theta)$ of $f(t)$, the Fourier transform of which is the energy spectrum $g(\nu)$ of the source. If the modulation frequencies are quite smaller than the spectral width $\Delta\nu$ of the source, $M(\tau - \theta)$ remains practically equal to $M(\tau)$ in the whole domain where $\Gamma(\theta)$ is not negligible.

That yields

$$\langle c_k^2 \rangle = \int_{-\infty}^{+\infty} L_k(t-\tau) P_{\text{in}}(\tau)\, d\tau \tag{10}$$

where $P_{\text{in}} = M(\tau)^2 \langle f^2 \rangle$ is the input power and where L_k is the impulse power response

$$L_k(t) = \frac{1}{\Gamma(0)} l_k(t) \int_{-\infty}^{+\infty} l_k(t-\theta)\Gamma(\theta)\, d\theta. \tag{11}$$

The frequency power response is the Fourier transform of (11):

$$L_k^t(\nu) = \frac{1}{\Gamma(0)} \int_{-\infty}^{+\infty} g(\nu') \exp - i[\beta_k(\nu') - \beta_k(\nu' - \nu)]z\, d\nu' \tag{12}$$

and another form of (11) is

$$L_k(t) = \frac{1}{\Gamma(0)} \iint_{-\infty}^{+\infty} g(\nu') \cdot \exp i[2\pi\nu t - (\beta_k(\nu') - \beta_k(\nu' - \nu))z]\, d\nu\, d\nu'. \tag{13}$$

As the spectrum function is nearly always zero except around $\nu = \pm\nu_0$, these integrals may be calculated using a Taylor expansion of $\beta(\nu)$ near ν_0. The details of the calculation are in Appendix B. Hereafter, the simpler results are given. The frequency response may be written

$$L_k^t(\nu) = \cos(2\pi^2\beta_k''\nu^2 z) \exp\left(-iz\frac{2\pi\nu}{V_k}\right) G(2\pi\beta_k''\nu z) \tag{14}$$

where $\beta_k'' = d^2\beta_k/d\omega^2$ at $\nu = \nu_0$, V_k is the group velocity at $\nu = \nu_0$, and where $G(\theta)$ is the complex envelope of the correlation function (see B13 and B14). The expression (14) is valid when

$$z \gg z_0 = (12\pi\beta_0''\nu_0\Delta\nu^3)^{-1/2}. \tag{15}$$

With longer fibers such as

$$z > z_1 = 10/(\beta_k''\Delta\nu^2) \tag{16}$$

the frequency response becomes simpler:

$$L_k^t(\nu) = \exp - iz\frac{2\pi\nu}{V_k} G(2\pi\beta_k''\nu z) \tag{17}$$

and it is associated to the following impulse response:

$$L_k(t) = \frac{1}{\pi|\beta_k''|z} \frac{1}{\Gamma(0)} g\left(\nu_0 + \frac{t - z/V_k}{2\pi\beta_k'' z}\right) \tag{18}$$

within less than 1-percent error. The expression of the impulse response corresponding to (14) and (15) is much more complex (Appendix B).

The form (18) has already been proposed by previous workers, without demonstration, on the basis of an intuitive interpretation [1]. Condition (16) shows its use is not always possible.

TABLE I

$\Delta\lambda$ (Å)	300	30	3	0,3
z_0 (m)	$2\ 10^{-3}$	$6\ 10^{-2}$	2	60
z_1 (m)	1	10^2	10^4	10^6
$\Delta\nu$ (Ghz)	10^4	10^3	10^2	10

Table I gives the values of z_0 and z_1 for different kinds of sources, characterized by their wavelength spectrum width $\Delta\lambda$, for a central wavelength of 0.85μm, and for silica fibers with $\beta'' \simeq 0.510^{-25}\ \text{m}^{-1}\text{s}^2$ [5]. The third line is the spectrum width $\Delta\nu$ in gigahertz: The frequency modulation must be quite smaller than $\Delta\nu$ so that the linear theory is valid.

For a 1-km-long fiber, we may use the simple expression (18) only if $\Delta\lambda \geq 30$ Å. However, when the spectrum is a set of several lines of width $\delta\lambda$ (as it is often the case for GaAs lasers, with $\delta\lambda \simeq 3$ Å or less), we must take the value of z_1 corresponding to $\Delta\lambda = \delta\lambda$. The value $z_1 = 10^4$ m for $\Delta\lambda = 3$ Å means a 10-percent error with our 1-km-long fiber.

The validity of the more complex expression (14) is more extended. However, practically, we must have $z > 100z_0$, and even this expression must not be employed for very pure sources such as some DFB lasers except in case of very long fibers. But neglecting the mode coupling on such lengths is not possible, whatever its origin may be: defects of the fiber or connections between partial links.

Finally, we come to the responses of the whole fiber. If we call p_k the fraction of the total input power which is launched in the kth mode ($\sum p_k = 1$), the total responses are

$$L(t) = \sum p_k L_k(t) \quad \text{or} \quad L^t(\nu) = \sum p_k L_k^t(\nu) \tag{19}$$

which may be approximated

$$L_a(t) = \sum p_k\, \delta\left(t - \frac{z}{V_k}\right)$$

or

$$L_a^t(\nu) = \sum_k p_k \exp - iz\frac{2\pi\nu}{V_k} \tag{20}$$

when the temporal details of scale $2\pi\beta''\Delta\nu z$ are neglected. This well-known result is more usually established ignoring the incoherence of the source and assuming the conclusions of Section II, i.e., the modulated modes are independent in power.

If we assume that all the modes are associated with the same spectrum function $g(\nu)$, we may simply connect the actual responses (19) with the approximate ones (20). As all the β_k'' are nearly equal and may be reduced to the part β'' due to the only material dispersion effect [5], we may write

$$L^t(\nu) = L_a^t(\nu)G_0(\nu) \quad \text{or} \quad L(t) = L_a(t) * g_0 \tag{21}$$

with

$$G_0(\nu) = \begin{cases} \cos (2\pi\beta''\nu^2 z) G(2\pi\beta''\nu z), & \text{if } z \gg z_0 \quad (22a) \\ G(2\pi\beta''\nu z), & \text{if } z > z_1 \quad (22b) \end{cases}$$

and if $z > z_1$

$$g_0(t) = \frac{1}{\pi|\beta''|z} \frac{1}{\Gamma(0)} g\left(\nu_0 + \frac{t}{2\pi\beta'' z}\right). \quad (23)$$

IV. Conclusion

We have studied some aspects of the theory of the power in optical fibers, in the case of modulated waves. As in the case of steady waves, the normal modes keep being independent in power, but only because they are usually excited in a narrow spectrum in fibers with low dispersion; there might be some trouble with LED in glass fibers without cladding.

With incoherent sources, it was understood that the output power is a linear function of the input power when the modulation frequency remains below the spectrum width. Then, it becomes possible to calculate the impulse response and the frequency response as a summation over the normal modes. We did assuming no mode coupling; we found some classical results and we proved some intuitive ones. We pointed out their validity conditions and we showed how to correct them in more general conditions.

Appendix A

We may write

$$c_k' - c_k = \int_0^\infty d\nu \sum_p A_{pk}\xi_p \exp i(2\pi\nu t - \beta_p z) \quad (A1)$$

with

$$A_{pk} = [\{e_p \times h_k^{0*} \cdot u\} - \{e_p^0 \times h_k^{0*} \cdot u\}]/N_k^0. \quad (A2)$$

In the case of metallic waveguides, the space coordinates and the frequency are separated variables in the modal field components and the A_{pk} are rigorously zero. For our dielectric waveguides, we have $A_{pk} = 0$ only in the zeroth-order approximation where the variations of the e_p near ν_0 are neglected. In the first-order approximation, we obtain

$$A_{pk} = (\nu - \nu_0)\{\partial e_p^0/\partial\nu \times h_k^{0*} \cdot u\}/N_k^0. \quad (A3)$$

It is difficult to discuss the behavior of such an expression without the help of intuition. Since $\partial e_p^0/\partial\nu$ and e_p^0 have the same spatial frequencies, we may think that the A_{pk} decrease when one increases the difference between the spatial frequencies of modes (p) and (k); inversely, we get a maximum for $p = k$. We checked these predictions in the case of the dielectric plate waveguide. Thus, when the modal amplitudes do not strongly change from one mode to the next one, we may limit the summation $\sum_p$ in (A1) to the modes near the mode (k). At the limit, considering only $p = k$, we should obtain a valuable order of magnitude.

Now, we assume that our modes are transverse. This hypothesis is a good approximation whenever there is little change from the core refractive index to the cladding index, as usual in practical fibers [5]. Then one can define a mode impedance z_k (equal to $\omega\mu/\beta_k$ for TE modes or $\beta_k/\omega\varepsilon$ for TM modes) such as $h_k = u \times e_k/z_k$ and, for free modes (e_k real and $N_k = \{e_k^2/z_k\} = 1$),

$$A_{kk} \simeq (\nu - \nu_0)\left\{\frac{\partial e_k^0}{\partial\nu}\frac{e_k^0}{z_k}\right\} = -\frac{1}{2}(\nu - \nu_0)\left\{\frac{\partial}{\partial\nu}\left(\frac{1}{z_k}\right)e_k^{02}\right\}. \quad (A4)$$

The mode impedance is constant over the cross section for TE modes; we may neglect its variations for TM modes. Then

$$A_{kk} \simeq \frac{1}{2}(\nu - \nu_0)\frac{1}{z_k}\frac{\partial z_k}{\partial\nu}\{e_k^{02}/z_k\} = \pm\frac{1}{2}\frac{\nu - \nu_0}{\nu_0}\beta_k\frac{\partial}{\partial\nu}\left(\frac{\nu}{\beta_k}\right). \quad (A5)$$

One obtains c_k by changing A_{kk} into 1 in (A4). Thus the upper limit of A_{kk} in the domain of existence of ξ_k gives the order of magnitude of the error

$$\left|\frac{c_k' - c_k}{c_k}\right| \sim \frac{1}{4}\frac{\delta\nu}{\nu_0}\left|\beta_k\frac{\partial}{\partial\nu}\left(\frac{\nu}{\beta_k}\right)\right| \quad (A6)$$

which is equivalent to (7) in the main text.

Appendix B

With $F(\nu,\nu') = \exp - iz[\beta(\nu') - \beta(\nu' - \nu)]$ (we omit the subscript k for simplicity) the frequency response may be written as follows:

$$L^t(\nu) = \frac{1}{\Gamma(0)}\int_0^\infty g(\nu')[F(\nu,\nu') + F(\nu,-\nu')]\,d\nu'. \quad (B1)$$

We assume that ν and $u = \nu' - \nu_0$ are small enough to allow the use of limited Taylor expansions of $\beta(\nu')$ and $\beta(\nu' - \nu)$:

$$F(\nu,\nu') = \exp - iz[2\pi\beta_0'\nu + 4\pi^2\beta_0''u\nu - 2\pi^2\beta_0''\nu^2 + \cdots] \quad (B2)$$

$$F(\nu,-\nu') = \exp iz[\beta(\nu') - \beta(\nu + \nu')] = \exp - iz[2\pi\beta_0'\nu + 4\pi^2\beta_0''u\nu + 2\pi^2\beta_0''\nu^2 + \cdots] \quad (B3)$$

where β_0' and β_0'' are the derivatives of β with respect to $\omega = 2\pi\nu$, at $\nu = \nu_0$

The cutoff frequency ν_m of $L^t(\nu)$ roughly is the frequency for which the phase of the integrand in (B1) is changed by 2π when ν' sweeps the width $\Delta\nu$ of the spectrum:

$$4\pi^2\beta_0''\Delta\nu z\nu_m \sim 2\pi. \quad (B4)$$

The second-order Taylor expansion is valid only if the third-order terms remain negligible for $\nu = \nu_m$ and $u = \Delta\nu/2$. An examination of these terms leads to the condition

$$z^2 \gg (6\beta_0''^3\Delta\nu^3/\beta_0''')^{-1} \quad (B5)$$

or, more roughly (with $\beta_0''' \sim \beta_0''/2\pi\nu_0$),

$$z^2 \gg z_0^2 = (12\pi\beta_0''^2\nu_0\Delta\nu^3)^{-1}. \quad (B6)$$

When this condition is satisfied, the approximate expression of $L^t(\nu)$ derived from the combination of (B1), (B2), and (B3), is valid throughout the frequency domain $|\nu| < \nu_m$. Out of this domain, it yields rapidly negligible values as the exact expression does. Thus it may be used in the Fourier transform giving $L(t)$ from $L^t(\nu)$

$$L(t) = \frac{2}{\Gamma(0)} \int_0^\infty d\nu' \int_{-\infty}^{+\infty} d\nu \cdot \exp\left[2i\pi\nu\left(t - \frac{z}{V_0} - 2\pi\beta_0''zu\right)\right] \cos(2\pi^2\beta_0''\nu^2 z) g(\nu')$$

where V_0 is the group velocity. We can integrate over ν exactly:

$$L(t) = \frac{2}{\Gamma(0)(2\pi|\beta_0''|z)^{1/2}} \cdot \int_0^\infty \sin\left[2\pi^2|\beta_0''|z\left(\frac{t - z/V_0}{2\pi\beta_0''z} - u\right)^2 + \frac{\pi}{4}\right] \cdot g(\nu')\, d\nu'. \qquad \text{(B7)}$$

This result is valid as long as $z \gg z_0$, where z_0 is given by (B5) or (B6). But if z is even larger, one may use the identity (in terms of distribution)

$$\lim_{p\to\infty} \sqrt{\frac{2p}{\pi}} \sin (\text{or} \cos)\, px^2 = \delta(x). \qquad \text{(B8)}$$

Then with

$$\nu_1 = \nu_0 + \frac{t - z/V_0}{2\pi\beta_0''z} \qquad \text{(B9)}$$

we obtain

$$L(t) = \begin{cases} 0, & \text{if } \nu_1 < 0 \\ \dfrac{1}{\pi|\beta_0''|z} \dfrac{g(\nu_1)}{\Gamma(0)}, & \text{if } \nu_1 > 0. \end{cases} \qquad \text{(B10)}$$

We still have to state the conditions of validity of this very simple result. The spectrum function $g(\nu)$ is often considered as a Gaussian function like $\exp -4(\nu - \nu_0)^2/\Delta\nu^2$ for $\nu > 0$ ($\Delta\nu$ being the $1/e$ width), or a sum of such functions.

For all of them, we always have $\Delta\nu \ll \nu_0$, and then, the integration in (B7) may be carried from $-\infty$ to $+\infty$, leading to an exact analytical result which allows one to check the accuracy of (B10). One finds that the highest absolute error occurs at $\nu_1 = \nu_0$ (i.e., the maximum of the Gaussian function) and that it corresponds to a relative error smaller than 10^{-n} if

$$z > z_1 = 10^n/(\pi^2\beta_0''\Delta\nu^2). \qquad \text{(B11)}$$

We also may have simple expressions for the frequency response. We obtain from (B1), (B2), and (B3)

$$L^t(\nu) = \cos(2\pi^2\beta_0''\nu^2 z) \exp\left(-iz\frac{2\pi\nu}{V_0}\right) G(2\pi\beta_0''\nu z) \qquad \text{(B12)}$$

where $G(x)$ is derived from the spectrum function:

$$G(x) = \frac{2}{\Gamma(0)} \int_0^\infty \exp - 2i\pi x(\nu' - \nu_0) g(\nu')\, d\nu'. \qquad \text{(B13)}$$

This function is related to the correlation function by

$$\Gamma(\theta) = \Gamma(0)\, \text{Re}\, [G(\theta) \exp - 2i\pi\nu_0\theta]. \qquad \text{(B14)}$$

It is generally complex valued, except if $g(\nu)$ is symmetrical with respect to ν_0. It starts from $G(0) = 1$, and its modulus is a nonoscillating decreasing function with a characteristic time $1/\Delta\nu$.

Expression (B11) corresponds to (B6); it holds when the same condition $z \gg z_0$ is satisfied. With the more stringent condition $z > z_1$, it may be simplified, since for the cutoff frequency ν_m given by (B3) one has

$$2\pi^2\beta_0''\nu^2 z = (\pi/2)(z_1/z)10^{-n} \ll 1$$

which allows one to write simply

$$L^t(\nu) = G(2\pi\beta_0''\nu z) \exp - iz\frac{2\pi\nu}{V_0}. \qquad \text{(B15)}$$

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